

Using Systems Tools to Construct Knowledge of Linear Equations

Laura Stepanek and Ron Michalak
Orange Grove Middle School
Catalina Foothills School District
Waters Grant Project

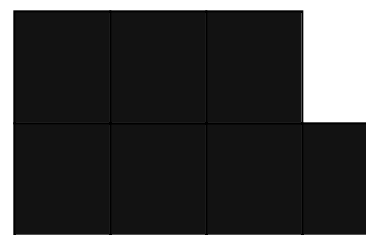
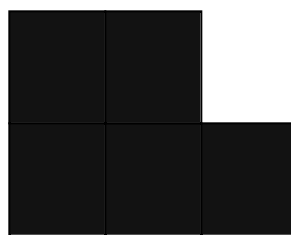
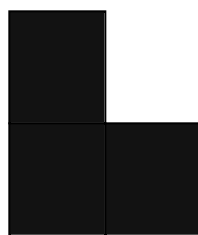
Number of min

1

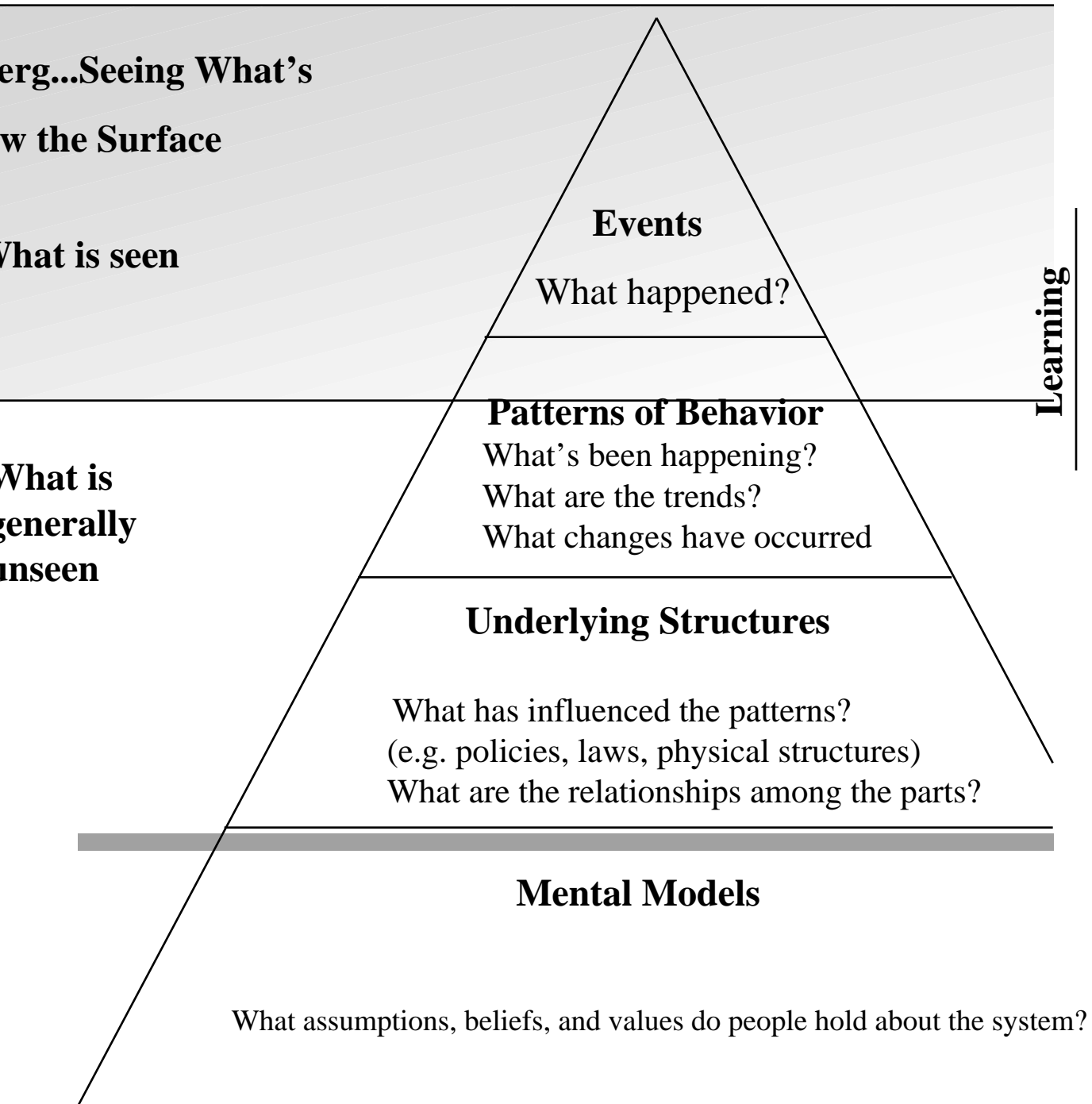
2

3

Gal of water



- How many gallons will there be in the fourth minute?
- How many gallons will there be in the 20th minute?
- Create a table of values
- Write the equation for this story
- How long will it take to accumulate 163 gallons?
- How many gallons will there be if no time has gone by?
- What is the change per minute



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Use this equation to answer the following questions:

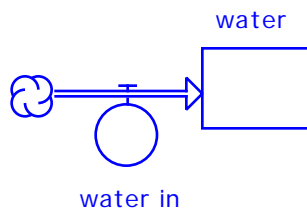
$$Y = -3x + 2$$

1. What is the slope of this line? _____
 2. What is the y intercept of this line? _____
 3. Why is this line linear? _____
-
4. Define y intercept. _____

5. Explain $y=mx+b$

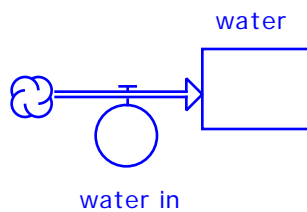
Given the stella model, describe the graph using x as a maximum of 20. Remember to label axis, y intercept, slope of line, and final point on the line.

1. Description

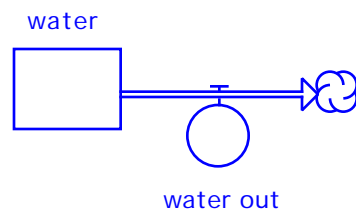


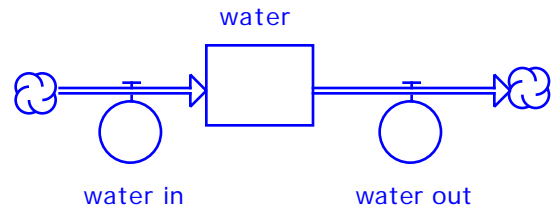
Write the equation for each stella model:

1. Equation:



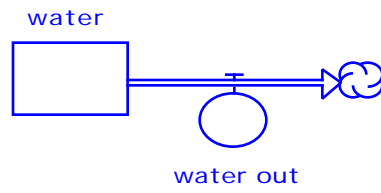
2. Equation:





Equation:

Given the model and equation above, fill in this model which will have the same equation.

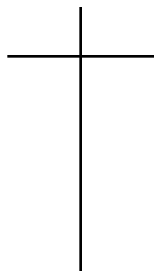


Name _____
Per. _____
Date _____

Study Guide: Book 2

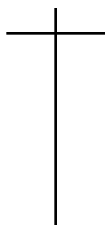
Write the equation in y intercept form, draw a graph and stella model for each of the following table of values.

1.



equation:
stella model:

2.



equation:
stella model:

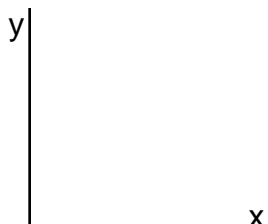
Write the equation for each sentence. Where y is the term number and x is the term number.

1. The term is three times the term number plus one.
2. The term is one fifth the term number.
3. The term is eight more than the term number.

For each sequence, make a table, draw a graph, and write an equation. Then predict the 10 term.

1. 3,7,15,31

2. 34,32,30,28



Name _____
 Per. _____
 Date _____

Quiz: Book 2
 (32 points)

Write the equation in y intercept form, draw a graph and stella model for each of the following table of values. Then predict the 150 term and write it in the table of values. (4 points each)

1.

x	y
1	6
2	10
3	14
4	18



equation:
 stella model:

2.

x	y
1	4
2	8
3	12
4	16



equation:
 stella model:

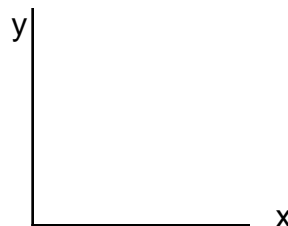
Write the equation for each sentence. Where y is the term and x is the term number. (1 point each)

- The term is two times the term number plus seven.
- The term is one eighth the term number.

For the sequence, make a table, draw a graph, and write the equation in y intercept form (3 points).

sequence: 33, 30, 27, 24

x	y



Use this equation to answer the following questions: (1point each for #1-4)

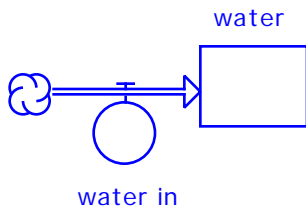
$$Y = -5x + 1$$

1. What is the slope of this line? _____
 2. What is the y intercept of this line? Give the x and y coordinates:) _____
 3. Why is this line linear? _____
-
4. Define y intercept. _____

5. Explain $y=mx+b$ (3 points)

Given the stella model, describe the graph using x as a maximum of 20. Remember to find the equation, y intercept, slope of line, and final point on the line. (4 points)

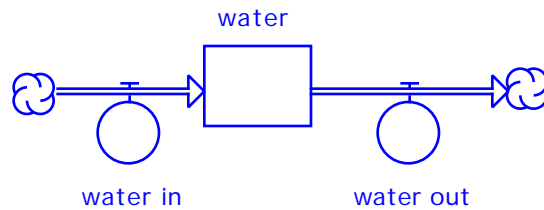
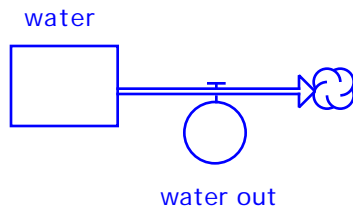
1. Description



Write the equation for each stella model (1 point each).

1. Equation:

2. Equation:



Solve the problem and write if the graph would be increasing, decreasing, or a horizontal line (2 points each)

Water in = blue (b)

Water out= red (r)

1. $3r + 5b$

2. $4r + b$

3. $123r + 123b$

Exploring the Equation D=RT

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System Dynamics Lesson Plan

D=RT

Curriculum: 7th/8th Grade Beginning Algebra

Lesson Objectives:

1. Students will be able to create a table, write an equation, and graph the equation $D=RT$.
2. Students will be able to identify the difference between constant and variable slope.
3. Students will be able to solve for one of the variables in the equation $D=RT$.
4. Students will be able to apply prior knowledge of rates and practice changing them in the Speed Machine STELLA simulation.

Systems Concept(s): Change over time, patterns and trends, accumulations, interdependencies, structure generates behavior, and temporal and spatial boundaries.

Systems Tool(s) Behavior-over-time graph, stock flow diagrams, iceberg, and computer model.

Activities at a Glance: Activity 1: Speed Machines STELLA Computer Simulation. Students answer questions based on developing tables, looking for patterns, and finding relationships among the variables. They read and interpret the graphs and identify the differences between constant and varied slope of a linear function. Lastly, the students work with the concept of y-intercept and solving a system of equation problems with the Romeo and Juliet speed machine.

Debrief Option A: Systems modeling of $D=RT$. Discuss and understand the “iceberg” and how the equation $D=RT$ can be portrayed by the iceberg metaphor. In addition, students recreate the STELLA computer model for $D=RT$ that was used in the Speed Machine simulation.

Debrief Option B: Discuss and demonstrate how the formula $D=RT$ can be predicted mathematically. Students are shown the stock and flow diagram below the Speed Machine simulation to help them understand the relationship among the variables of distance, rate, and time that generated the tables and graphs in the student packet.

Activity 2: Radio Controlled Cars. The students drive radio controlled cars certain distances, measured in inches, and time the cars in seconds. The students then have to calculate the rate of speed for each car in terms of feet per second and convert them to miles per hour.

Time Allotment: Activity 1: Three 42 minute periods to introduce and run the simulation.

Debrief Option A: Two class periods of 42 minutes having students create and develop the models themselves.

Debrief Option B: One to two class periods of 42 minutes. Depending on how familiar students are with stock/ flow diagrams.

Activity 2: Two or three class periods, 42 minutes each. One period to have students collect data and another class period or two to review rates and assess handout.

Integration Area(s):

Materials Needed For Lesson:

Materials: Activity 1: D=RT packet/ handouts (one per student; see attached)
Debrief Option A: Computer packet from Activity 1, systems' "iceberg" poster
Debrief Option B: Computer packet from Activity 1.

Activity 2: 3 radio controlled cars, batteries for cars, 3 rolls of masking tape, 3 stop watches, 8 yard sticks, handouts (one per student; see attached)

Media: Activity 1 and Activity 2: STELLA 5.0 software, D=RT Speed Machine simulation.

Equipment: Activity 1, Debrief Option A and Debrief Option B: Projection system (overhead with LCD panel or computer projector) with computer able to connect to one of these. One color Macintosh computer for every two students.

Preparation: Activity 1, Debrief Option A, and Debrief Option B: Copy handouts, install software on necessary machines, set up projection system.

Activity 2: Copy handouts and collect all materials/supplies.

Background Information:

1. Teacher: read the handouts. Need basic modeling experience or may have SD mentor present the debriefing options.
2. Students: prior knowledge of graphing in the first quadrant for Activity 1. Students should be familiar with rates for activity 2.

Activity(ies): Activity 1: Talk to students about the upcoming lesson (learning about D=RT and understanding the concept of what the variables represent).

1. In the computer lab, use the computer and the projector to show what the simulation looks like and explain the graph axes.
2. Hand out the D=RT packet, and go over the directions on the first speed machine.
3. Show students how to change the slide bars and how to run the simulation. They can do this at the same time on their own computers.
4. Have students work on the packets either individually or with a partner (depending on the number of computers)

Activity 2: Review the concept of rates and the conversion of units in the context of a radio controlled car.

1. Have students get into 3 equal groups and pass out handout.
2. Go over the instructions on the handout. Make sure every student understands his/her role and that he/she will need to work with the group and share responsibilities. Go over measurement and remind students how to record data in the correct column.
3. Afterward, the students need to make sure that the data they collected was accurate and reasonable by comparing it with the other groups through group discussion.
4. The teacher can use one set of data and give an example showing students how to convert rates and show work.

Debrief/Closure: Activity 1: See attached sheets - Debrief Option A or Debrief Option B.
Activity 2: Correct radio controlled car worksheet by having students give several examples of their answers.

2. Talk about the relationships among the variables of distance, rate, and time.
3. Have students identify and give examples, using the iceberg, of an event, pattern of behavior, underlying structure and mental model from handout. (If time, one from each car).

Assessment:

One assessment option is have students turn in their completed handouts and grade the students on their ability to see relationships and patterns. Furthermore, students can be assessed on the accuracy of their computations in answering the questions. Another option is to go over the handout as a class and grade and answer each question. Students will continue throughout the year to practice and apply the skills learned by creating tables, developing equations, and graphing. Finally, students will be solving for a variable on more traditional assessments later.

Sources:

Extension(s):

1. Motion detector activity where students create graphs kinesthetically with time and distance on the axes. With this activity, I continued reviewing graphing and taught the concept of a function.
2. Have students read a graph with time and distance labeled on its axes and write a story based on that graph.
3. Have students write their own story based on $\text{distance} = \text{rate} \times \text{time}$ multiplied by time. Have students switch stories and graph their partner's story.

Additional Resources For Follow-up Activities/Lessons:

Speed Machines Simulation Debrief Option A:
(Iceberg and student modeling of $D=RT$)

- Show students the iceberg
 - Events
 - Explain and give examples of events unrelated to Math e.g. event from the books, The Outsiders and The Giver which they had read in the past, e.g. the bell rings and a student is late for class, eat pizza on Saturday night, a student receives an 'F' on a test.
 - Have students look at page 1 of their completed Speed Machine packet.
 - Ask, "Looking at the first page of your packet, what would an event look like in Math within this unit? e.g. have traveled 50 miles after one hour. If students have trouble, give an example such as, "You are traveling 20 miles per hour, and after one hour we freeze time. How far have you gone?" This distance is the event.
 - Ask students to describe several other events at 20 miles/hour e.g. after three hours, we've traveled 60 miles.
 - Patterns
 - Ask a series of questions to elicit an understanding of patterns related to the events described previously:
 - What would show the pattern of these events? e.g. By linking the events over time, a pattern would emerge. Note: most students need additional questions in order to understand this. (see following questions)
 - Can you have a pattern with just one event? Answer: no, you need at least two events.
 - Look on the first page. What could show a pattern?
 - How many events do you need?
 - How would you explain the pattern of the 20 mile/hour events as compared to the 60 mile/hour events
 - What gives you a picture of the pattern? e.g. the graph. (Note: differentiate between the graph and the table. The table gives you the events, which can be used to deduce a pattern. The graph actually shows the pattern in a picture.)
 - Structure
 - Ask a series of questions while referring to the iceberg, to elicit an understanding of the mathematical structure that generates the patterns seen in the graphs in the packet
 - What is the rule that shows the relationship of distance to time? ($D=RT$)
 - What is the rule that shows the relationship of distance to rate? ($D=RT$)
 - Discuss how the formula $D=RT$ is a mathematical sentence explaining the relationships among the parts D, R, and T
 - Have students turn on computers and open a new model page in STELLA
 - Begin the next section of this debrief by explaining to students that to help increase their understanding of the relationships among distance, rate, and time that we are going to use a different structure, the structure of a STELLA computer model, to express those relationships. Tell students that they will be building the model themselves.
 - Begin with a review of the bathtub analogy and have students place a stock in the middle of their computer screen
 - Ask students if they know what is accumulating in the $D=RT$ equation - distance. If distance is accumulating then in the STELLA structure what should we name the stock?- D (distance).
 - Have students name the stock on their machines. Open the stock and have them determine that the initial value should be 0.
 - Follow the process above as students hook up a flow and discover that the flow will represent R or the rate. Have students open the flow and

put in a rate from one of their student packet sections e.g. 20 miles per hour.

- Next ask students what element is still missing - T (time). Explain how time is invisible in the STELLA structure and that the program multiplies the rate by the time units that the simulation runs.
- Have students run the simulation and then ask them what happened - can't tell because in STELLA you must have a graph to see the results of the structure in operation.
- Now explain to students that we are going to move back up the iceberg using STELLA and so we will need to create a graph to show the pattern of the structure we have built.
- Patterns (STELLA)
 - Have students move through the steps of setting up a STELLA BOTG for distance. Have them set the scale etc.
 - Next have students run the model and then compare their graphs to the appropriate one in their student packet. Explain that the patterns should be identical and if they are not then there is a problem in their model. Trouble shoot until all student models are reproducing the correct pattern.
 - Introduce the slider tool in STELLA and show students how they can use it to easily change the rate. They should then be able to reproduce all of the patterns of the Constant Rate Speed Machine.
 - Next, show the students how to have their models pause after 5 time units. Have them adjust the model's graph to show the rate as well as the distance. They should then be able to replicated all of the patterns of the Variable Rate Speed Machine.
- Events (STELLA)
 - Finally have students find specific events in their graphs (rate of 20 mph for 5 hours - the distance in 100 miles) and realize that these match the events listed in the chart in their student packet. (if time permits, the students can create their own STELLA tables)
- Conclude with a discussion as to whether the STELLA stock/flow structure does indeed replicate the patterns of the mathematical equation structure of $D=RT$.
- If time allows take students through the process of creating the models necessary for the Romeo and Juliet Speed Machine.

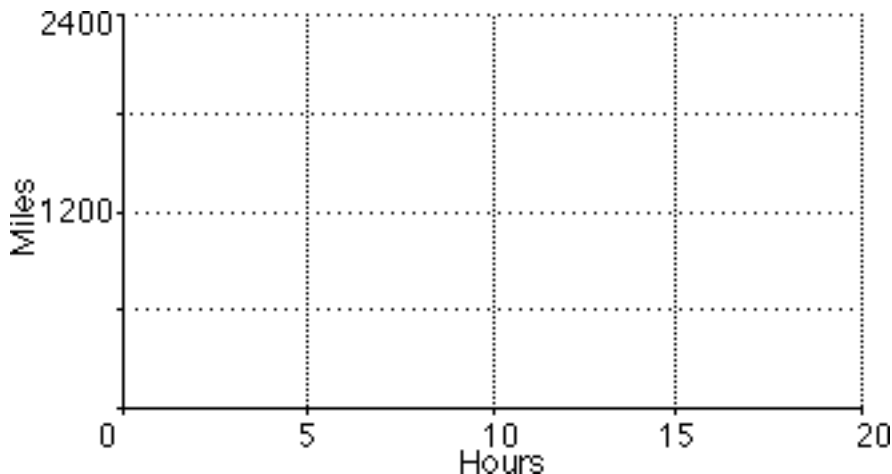
**Speed Machines Simulation Debrief Option B:
(without iceberg and student modeling)**

1. Begin by talking about making predictions.
 - A. How many students predicted the computer generated graphs correctly?
 - B. Why or why not? (“I used the formula so my predictions were exactly right.” or “I was just guessing so even though I knew that the slope would be greater as the speed went up, I didn't not know this exactly.”)
2. Ask how many students did the Giver simulation in 6th grade. (You could also reference the Krill model)
 - A. Talk about the difference between making predictions about abstract vs. concrete situations. e.g. when they did the giver simulation in 6th grade, they were making predictions about things like feelings and emotions. Now we are using a formula. When the students did the Krill model simulation, the students did not know exactly how many krill there were in the ocean and so predicting what would happen was more difficult as they had to rely on the strategies of sampling and estimation.
 - B. Ask the students how making predictions using a formula would be different from making prediction without using an equation.
3. Write the formula $D=RT$ on the board.
 - A. Talk about the equal sign, how one side of the formula exactly equals the other or the concept that equals means “the same as”. A formula can show a relationship or pattern between different parts of a system. This formula shows how distance is related to rate and time.
4. “ What’s accumulating?” You may need to explain the word accumulation. Answer: distance.
5. Project the model onto the wall.
 - A. Go down a layer to show the model
 - B. Explain the bathtub analogy
 - C. Explain the stock/flow diagram as a visual or different structure to represent the formula $D=RT$
 - D. Explain the stock of distance
 - E. Input the speed and show how as the model runs, miles are pumped into the stock of distance.
6. Go back up a layer to Speed Machine 1: Constant Rate. Run it once and then explain how the graph exactly equals the rate multiplied by time so we can predict exactly what will happen.
7. Go through the rest of the handout to make sure students understand the patterns relating to speed, distance, and time. Project the model as you are doing this. Generate different graphs using the model as you are talking about the equation $D=RT$.

Simulation Exploring the Formula, Distance = Rate X Time (D=RT)

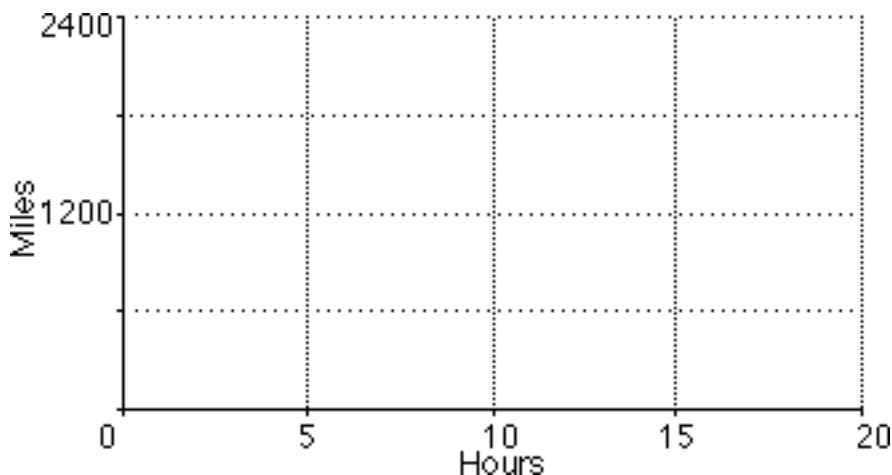
I. Constant Rate Machine

- Predict what will happen if the bus travels 20 mph, 40 mph, 60 mph, and 120 mph for 20 hours. You will need to draw four lines on the graph below.



- READ THE INSTRUCTIONS!! then run the bus at 20 mph and fill in the table below as the simulation is running. Now run the bus for 40 mph, 60 mph, and 120 mph filling in the chart below during each run. Four lines should now show on the graph. Record them on the graph below.

	5 hours	10 hours	20 hours
20 mph			
40 mph			
60 mph			
120 mph			



Were your predictions exactly correct? _____

If not, why?

3. Why is 2400 the maximum for y axis?

4. Write an equation in terms of the axes on the graph for a bus going:

20 mph? _____

40 mph? _____

60 mph? _____

120 mph? _____

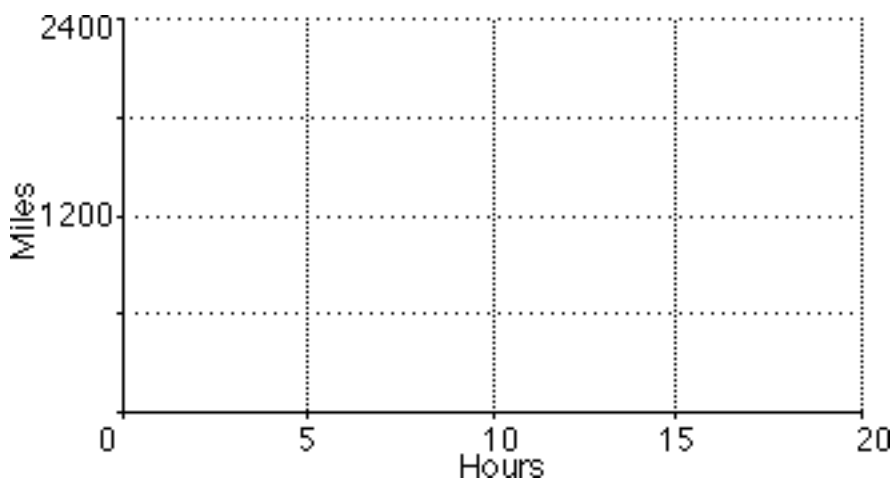
5. Compare the distances for 20 mph at 5 and 10 hours. Do you see a relationship between these two distances? If yes, what? _____

Do the same comparison for 40 mph at 5 and 10 hours. What is the relationship?

Do the same for 60 mph at 5 and 10 hours. What is the relationship?

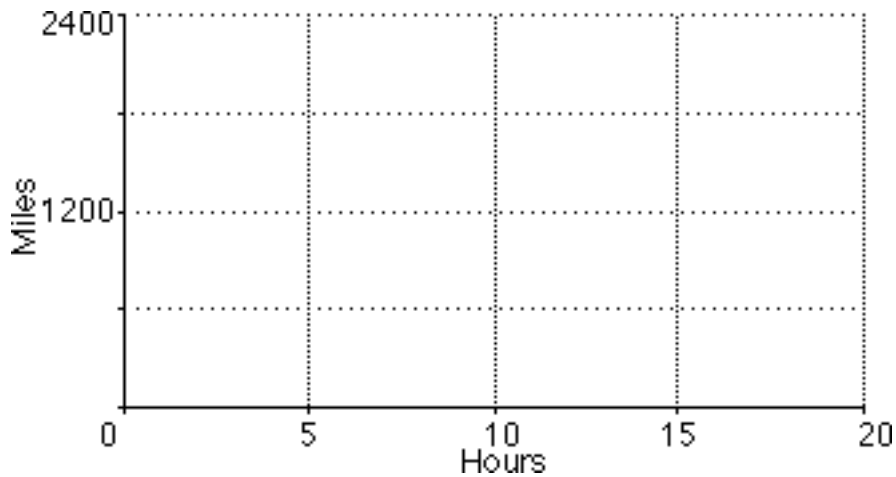
Given this relationship, explain in general terms what happens to distance over time.

6. BOMB THE BUS! Predict on the graph below what will happen with the distances at 30 mph, 60 mph, and 90 mph in relation to one another.



7. Run the simulation for 30 mph, 60 mph, and 90 mph and record your results below during each run. Record your graph below. Make sure to label the lines with the speed.

	5 hours	10 hours	15 hours	20 hours
30 mph				
60 mph				
90 mph				



8. What are the patterns in the table? Why do the patterns exist? Hint: think about time and speed relationships.

9. Describe all the patterns you find with these lines on this graph. Hint: think about time and speed relationships. If you need more room to write, attach a separate sheet of paper.

BOMB THE BUS, turn off this sector and RETURN to the first screen

II. Variable Rate Machine

1. What do you think "variable rate" means? _____

2. How do you think this machine/ screen will be different than the constant rate?

3. Read the instructions, FIRST. Run the simulation at 30 mph across the entire screen. What does the red line measure? _____
 What does the blue line measure? _____
4. Was the rate constant or varied? _____
 How does the graph show you? _____
5. Did the distance (red line) increase or decrease? _____
 Was it by the same amount each 5 hours? Hint : you can use the table you created for question 7 above to help you: _____
 Why or why not? _____

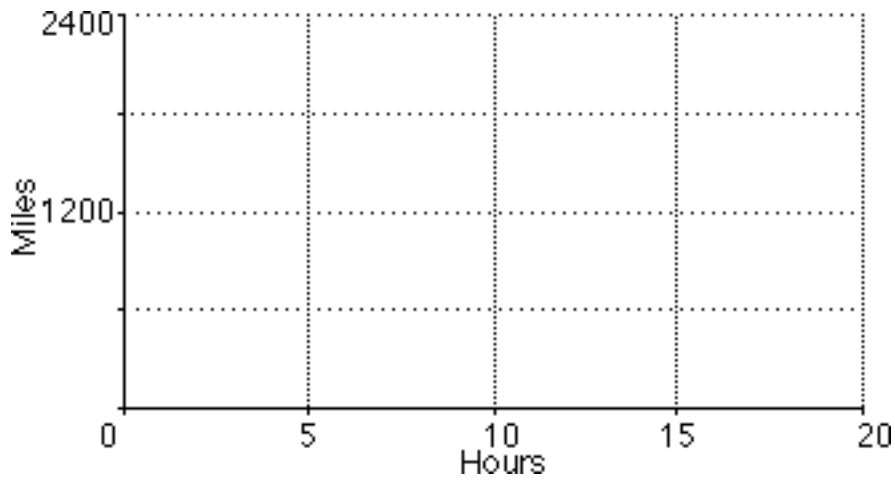
6. How far would the bus go at 30 mph after 25 hours? _____
 How did you get the answer? _____

7. BOMB THE BUS and Run the simulation with the scenario below. Fill in the table on the next page during the run.

TIME	SPEED
0-5 hours	30 mph
5-10 hours	60 mph
10-15 hours	90 mph
15-20 hours	120 mph

Time	Speed	Dist in 5hrs	Tdtal Dist.
0-5 hrs (5hrs)	30 mph		
5-10 hrs (5hrs)	60 mph		
10-15 hrs (5hrs)	90 mph		
15-20 hrs (5hrs)	120 mph		

Copy your results on the graph below and label.



8. Describe what happened to the red line (distance) compared to the lines on the graph shown in question 7 of the constant rate machine.

Why do you think they are different? Why is the line on this graph curved?

9. What is the difference in the total distances between the intervals

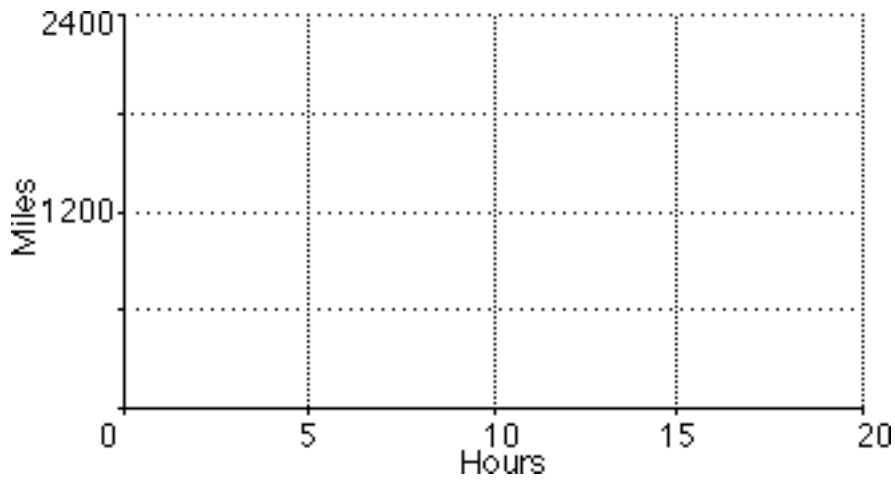
0-5 hours and 5-10 hours? _____

5-10 hours and 10-15 hours? _____

10-15 and 15-20 hours? _____

10. Why are they different? Describe and explain the pattern?

11. **BOMB THE BUS.** Run the bus for the first 5 hours at 120 mph. The second 5 hours run the bus at 90 mph. The next 5 hours, run the bus at 60 mph. Finally, run the bus at 30 mph for the last 5 hours. Copy the graph below.



12. How is this graph similar to the graph that had the bus run 30 mph, 60 mph, 90 mph, and then 120 mph? _____

13. How are the the graphs of the buses different? _____

14. What is your explanation for the differences?

15. During this run, the blue line (speed) was decreasing. Why didn't the red line (distance) decrease? _____

BOMB THE BUS, turn off this sector and **RETURN** to the first screen

III. Romeo and Juliet Machine

1. Read the instructions
2. Run the simulation (try to have Romeo meet Juliet before the 20 hours is over. Record all your efforts on a chart like this (1 chart per try):

Try 1:

	Starting Distance	5 Hours	10 Hours	15 Hours	Distance @ 20 hrs.	Did they meet in L.A.?
Juliet						
Romeo	0					

Try 2:

	Starting Distance	5 Hours	10 Hours	15 Hours	Distance @ 20 hrs.	Did they meet in L.A.?
Juliet						
Romeo	0					

Try 3:

	Starting Distance	5 Hours	10 Hours	15 Hours	Distance @ 20 hrs.	Did they meet in L.A.?
Juliet						
Romeo	0					

Try 4:

	Starting Distance	5 Hours	10 Hours	15 Hours	Distance @ 20 hrs.	Did they meet in L.A.?
Juliet						
Romeo	0					

Try 5:

	Starting Distance	5 Hours	10 Hours	15 Hours	Distance @ 20 hrs.	Did they meet in L.A.?
Juliet						
Romeo	0					

Try 6:

	Starting Distance	5 Hours	10 Hours	15 Hours	Distance @ 20 hrs.	Did they meet in L.A.?
Juliet						
Romeo	0					

Name _____

Date _____

Period _____

D=RT Recording Worksheet with Radio Controlled Cars

Instructions:

The class will be in three groups. Each group will have 10 minutes to collect the data and complete the cart/table for each car. The group needs to work together to make sure that you are able to finish in the time limit. The group need to share and rotate the following roles:

- timer
- measurer (2 people)
- driver (need to alternate)
- pit crew (people who ask for help or get materials)
- taper (person responsible for marking and cleaning up the tape before rotating)
- recorder (everyone)

The rotation of stations is: Black 11 to Yellow 7 to Orange 49. You may use a calculator to figure out the distances and rates. The conversions you need are in the table. Make sure your answers are labeled.

Type of Car	Trials	DISTANCE 12 inches = 1 foot	TIME (seconds)	RATE (feet per second i.e ft./sec.)	MPH 5280 ft=1m 60 sec=1min 60min=1hr.	Average MPH (for the 2 trials)
Black 11	1	160 in. or ()ft.				
	2	160 in. or ()ft.				
Yellow 7	1	80 in. or ()ft.				
	2	80 in. or ()ft.				
Orange 49	1	120 in. or ()ft.				
	2	120 in. or ()ft.				

Questions:

1. How long will it take each car, using the average mph, to get to Phoenix? Phoenix is 120 miles from Tucson. Show all work and label answers.
2. The radio controlled cars batteries last a total of 2.5 hours. If you were to run it continuously at the average mph, how far would each car go? Again, show all work and label answers.
3. If given the distance and the time, how would you figure out the rate?
4. If given the time and the rate, how would you figure out the distance?
5. If given the rate and the distance, how would you figure out the time?
6. On a scale of 1-10 (10 being the highest), how well did your group work together? Give reasons for your answer.

Waters Project - CFSD Assessments Evaluation of Effectiveness and Results

Grade: 7th

CFSD Curriculum: Math- Book 2- linear equations, slope, y intercept,

Lesson Objective(s): 4C-A-M1-4- Compare a given prediction with the results of an investigation. 2C-B-M2-1 -Produce the rule (function) that explains the relationship (pattern) between the numbers when a change in the first variable affects the second variable (T-chart, two-row table, or input/output machine). 1C-F-M3-3- Demonstrate understanding of rate and unit rate. 2C-C-M3-1- Describe and use variables in a contextual situation. 2C-C-M3-3- Match an equation with a fitted line. 2C-B-M1-1- Write a rule to extend patterns.

System Concept(s) and Tools Used in Instruction: change over time, patterns and trends, accumulations, interdependencies, temporal and spatial boundaries. and behavior-over-time graphs, iceberg, computer models.

Extensions from Objective (if applicable): unit conversions, negative slope, exponents, scaling.

Method of Assessment: Computer modeling performance assessment

General Description of the Assessment (attach examples and student sample): students use STELLA to create model structures that generate the same patterns and events as the traditional math equation solving structure for any given linear equation of the form $y=mx + b$. Students then transfer their understanding and model structures of linear equations to the specific case of $D=RT$. (see student model)

Results:

- **data collected** - 100 percent of students demonstrated the objectives
- **interpretation of data** - objectives were demonstrated with teacher guidance during the model building performance activity. Data does not speak to what results would be achieved if students were asked to build the models independently.
- **What does this assessment show about the effect of ST/SD on students' achieving the lesson objective?** This assessment shows that ST/SD provides an alternate structure to the traditional math approach that allows students to explore, confirm, and visually demonstrate their understanding of the math concepts and skills required to achieve the lesson objectives related to linear equations.

Indicators of Effectiveness of this Assessment:

- **How effective was this assessment?**- acceptably effective as 100% of students demonstrated the objectives as stated.
- **What was the validity of this assessment** -acceptably valid as the results were

Indicators of Effectiveness of this Assessment:

- **How effective was this assessment?**- acceptably effective as 100% of students demonstrated the objectives as stated.
- **What was the validity of this assessment** -acceptably valid as the results were replicated in two different classes

Comments: e.g. Unintended results